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VOTING BY QUOTA AND COMMITTEE ♦
by
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D.P. 64

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1. INTRODUCTION

Problems of social choice frequently take the following form. There are n voters and a set $K = \{1, 2, \dots, k\}$ of issues. Society must choose (adopt) a subset of the set of issues. Assume that each voter has a linear order of the 2^k possible subsets that can be adopted. A voting scheme is a method for passing from n -tuples of these linear orders, called preference profiles, to sets of issues. A scheme is dictatorial if there is some voter whose preferred choice is always selected, no matter what the preferences of other voters. It is manipulable if, for some profile of individual preferences, a voter can obtain an outcome set that he prefers by misrepresenting his true preferences. The Gibbard-Satterthwaite Theorem says that if there is more than one issue to consider for adoption, then all nonmanipulable schemes are dictatorial¹. (An additional hypothesis is required: the range of the scheme must contain at least three sets of issues.)

For social choice problems concerning the provision of public goods, the Groves-Clarke mechanism (see; for example, Holmström [1979]) provides a response to the Gibbard-Satterthwaite Theorem. Under the hypothesis that each voter's preferences are of the transferable utility form, the Groves-Clarke mechanism is not manipulable; furthermore, it is the only nonmanipulable scheme on this domain of preferences.

In this paper we first present a scheme for selecting subsets of a set K of issues; it is called voting by quota. In the quota system with quota Q ($1 \leq Q \leq n$), the i^{th} voter ($1 \leq i \leq n$) votes a subset of the issues $A_i \subset K$, and the outcome is the set of issues which belong to at least Q of the sets A_i .²

Our results run parallel to those for the Groves-Clarke scheme. Provided that preferences are additively representable, voting by quota is nonmanipulable. Furthermore, with the addition of some natural and classical hypotheses, voting by quota is the only nonmanipulable scheme on the domain of additively representable preferences.

¹ See Schmeidler and Sonnenschein (1978) for a convenient proof.

² Quota voting is used explicitly by some clubs when each year the membership convenes to elect new members: A candidate for membership is elected if his name is checked on some predetermined (say Q) number of ballots. A reduced form of voting by quota operates in many legislative bodies. Elected representatives normally vote on a large number of issues. Each year these issues are known and it is not unusual for legislators to announce for all to hear which issues they favor and which they oppose. Although the issues are in practice confronted one by one, with, for example, support by two-thirds of the legislators needed for adoption, it is "as if" all legislators announced at once the issues which they support (this might follow some logrolling) and "as if" issues announced by two-thirds of the legislators are adopted.

Definition 1: The preference relation $>$ on 2^K is additively representable if there exists a function $U : K \rightarrow \mathbb{R}$ such that for all $B, C \subset K$,

$$B > C \text{ if and only if } \sum_{z \in B} U(z) > \sum_{z \in C} U(z).$$

Additive representability requires a degree of independence among issues. An important case in which the condition will be satisfied is when (a) each issue can be gauged according to a finite number (say n) of characteristics, (b) preferences over bundles of characteristics are represented by a differentiable function U , and (c) changes from a status quo \bar{x} are infinitesimal. In an obvious notation, if an issue with characteristics $(dx_1, dx_2, \dots, dx_n)$ and an issue with characteristics $((dx_1)', (dx_2)', \dots, (dx_n)')$ are both adopted, then the resulting change in utility is $\sum(dx_i + (dx_i)')(\partial U(\bar{x})/\partial x_i)$. Writing $\sum(dx_i)(\partial U(\bar{x})/\partial x_i)$ for the utility of the issue $(dx_1, dx_2, \dots, dx_n)$ yields an additive representation of preferences. (Interaction effects lead to the failure of additive representability. A conservative legislator may favor the passage of any one of five issues by themselves; however, the adoption of any two together might, in his opinion, represent an intolerable change from the status quo. Similarly, Professors Glutz and Smith individually have your vote, but if they are both elected it will tear your institute apart.)

It is easy to see that voting by quota is not manipulable on the domain of n -tuples of additively representable preferences. To see this, fix a profile of preferences and let S denote the set of issues that will be included in the outcome set, independently of how the i th voter votes. With additively representable preferences, i 's preferred subset of K is exactly the union of all those issues which he would like to see added to S . Under voting by quota, by truthfully reporting his preferred subset, these issues are added to S .

Beyond nonmanipulability over the profiles of additively representable preferences, voting by quota satisfies three desirable classical properties. First, it treats issues symmetrically (this is called neutrality). Neutrality means that each issue is given a free hand to emerge and that one issue is not favored over another simply because of its "name". Also, voting by quota is symmetric with respect to individuals (this is called anonymity). Anonymity formalizes the requirement of "one man one vote". Finally, voting by quota satisfies voter sovereignty; this is the condition that there is no subset of the set of issues that is barred from emerging.

A key result of this paper is a characterization of voting by quota. Consider methods for passing from n -tuples of individual preferences, as expressed by n -tuples of orderings of subsets of the set of issues K , to sets of issues. Voting by quota, which

depends only on the top ranked subsets (this represents an economy of form), is the unique method from this class which satisfies anonymity, neutrality, voter sovereignty, and nonmanipulability on the domain of additively representable preferences.³

Voting by quota is a salient representative in a larger class of decision-making processes. Under voting by quota Q , we consider for each issue the set of voters who favor it, and pass the issue if that set of voters is of size Q or larger. This is a particular way of describing what coalitions of voters are winning, in the sense that they can force an issue into the final outcome. In general, we can design voting methods by first describing a family of winning coalitions and then letting any issue be passed if and only if the set of the voters who favors it is winning. (We assume that if a coalition is winning, then all its supersets are.) The device of specifying voters and winning coalitions is standard in game theory, and such specification is usually known as a (monotonic) simple game, or a committee. Thus we call these methods voting by committee. Voting by quota is anonymous voting by committee, since whether or not a coalition is winning depends only on the number of voters in the coalition and not on their names.

Our characterization of voting by quota generalizes. When the anonymity requirement is dropped, voting by committee is the unique method for passing from n -tuples of preferences over sets of issues to sets of issues which satisfies neutrality, voter sovereignty and nonmanipulability on the domain of additively representable preferences. In fact, similar characterizations of voting by quota and voting by committee also hold for a larger domain of preferences that we call the semiadditive preferences.

The paper proceeds as follows. In Section 2, we introduce notation, definitions, and the main theorems. Section 3 is devoted to proofs. In Section 4 we prove that voting by quota or voting by committee are manipulable on any domain larger than that of semiadditive preferences. In Section 5 we study the efficiency properties of our voting methods. Finally, in Section 6 we show that our results can be partially extended to cover nonneutral voting procedures.

³ Notice that on the domain of additively representable preferences any triple of sets of issues is free. Thus, no Arrovian social welfare function can be defined on this domain, over which voting by quota is a perfectly well defined strategy-proof decision scheme. This should be a warning against misinterpretations of results by Satterthwaite, Kalai and Muller, and others, regarding the close connection between the possibility of defining Arrovian social welfare functions and strategy-proof decision schemes on a domain.

2. NOTATION, DEFINITIONS, AND MAIN THEOREMS.

The set of voters is $N = \{1, 2, \dots, n\}$. The set of issues is $K = \{1, 2, \dots, k\}$. We assume that n and k are at least 2.⁴ Subsets of K are denoted by A, B, B', S, T, \dots , with the empty set denoted by \emptyset . The i^{th} voter's preferences, denoted by $>_i, >'_i$, etc., is an (asymmetric) ordering on 2^K (the set of subsets of K).⁵ Let P denote the set of all orderings on 2^K . The class of additively representable preferences is denoted by PA . (See Definition 1).

The voting schemes we consider have each voter order the 2^k subsets of issues and for each n -tuple of such orderings produces a winning set of issues. Formally, a voting scheme is a function $f: P^n \rightarrow 2^K$. If a voting scheme depends only on the subset of K placed on top by each voter, then it is of the form $f: (2^K)^n \rightarrow 2^K$. Voting by quota and voting by committee are of this form.

Informally, a voting scheme will be called strategy-proof if it is always in the best interest of voters to reveal their actual preferences. The formal definition of strategy-proofness qualifies the above statement by introducing a relevant domain $T = T_1 \times \dots \times T_n$.

Definition 2. The voting scheme $f: P^n \rightarrow 2^K$ is manipulable on T at $(>_1, >_2, \dots, >_n) \in T$ by $i \in N$ via $>'_i \in T_i$ if $f(>_1, >_2, \dots, >'_i, \dots, >_n) >_i f(>_1, >_2, \dots, >_n)$.

As a shorthand we will sometimes write that f is manipulable on T at $(>_1, >_2, \dots, >_n)$ by i via $>'_i$, or simply that f is manipulable on T . Also, we will say that the voting scheme f is strategy-proof on T if it is not manipulable on T .

The following two conditions require that all voters are treated alike and all issues are treated alike.

Definition 3. The voting scheme f is anonymous if $f(>_1, >_2, \dots, >_n) = f(>_{\sigma(1)}, >_{\sigma(2)}, \dots, >_{\sigma(n)})$ for all $(>_1, >_2, \dots, >_n) \in P^n$ and any permutation σ of N .

Definition 4. The voting scheme f is neutral if $f(\mu(>_1), \mu(>_2), \dots, \mu(>_n)) = \mu(f(>_1, >_2, \dots, >_n))$ for all $(>_1, >_2, \dots, >_n) \in P^n$ and any permutation $\mu: K \rightarrow K$. $\mu(>_i)$ is the ordering obtained from $>_i$ by permuting the issues by μ .

⁴ Our main results can also cover the cases $n = 1$ or $k = 1$. We exclude these trivial cases for ease of exposition, since including them would require minor qualifications in some of our statements.

⁵ The restriction to strict preference is natural since K is finite.

Definition 5. The voting scheme f satisfies voter sovereignty if for all $A \subset K$ there exists $(\succ_1, \succ_2, \dots, \succ_n)$ such that $f(\succ_1, \succ_2, \dots, \succ_n) = A$.

Let $M(\succ)$ denote the best element of 2^K according to \succ ; $M(\succ)$ is defined by $M(\succ) \succ A$ for all $A \neq M(\succ)$.

Definition 6. The voting scheme $f : (P^A)^n \rightarrow 2^K$ is voting by quota if there exists Q between 1 and n such that for all $(\succ_1, \succ_2, \dots, \succ_n)$, we have $x \in f(\succ_1, \succ_2, \dots, \succ_n)$ if and only if $\#\{i \mid x \in M(\succ_i)\} \geq Q$. (The notation $\#L$ denotes the number of the elements in L .)

It is easy to check that voting by quota satisfies all the properties we just defined. Our first theorem also states that it is the only method to do so, when preferences are restricted to be additively representable.

THEOREM 1: A voting scheme $f : (P^A)^n \rightarrow 2^K$ satisfies anonymity, neutrality, voter sovereignty, and is strategy-proof on P^A if and only if it is voting by quota.

As we noted in the introduction, additive representability implies the absence of interaction effects among issues. We now introduce a weaker restriction on preferences that still prevents these effects. For $\succ \in P$, let $G(\succ) = \{x \in K \mid \{x\} \succ \emptyset\}$ be called the set of good issues. Its complement $B(\succ)$ is the set of bad issues for \succ .

Definition 7. We say that a preference relation $\succ \in P$ is semiadditive if for all $A \subset K$ and all $x \notin A$, $A \cup \{x\} \succ A$ if and only if $x \in G(\succ)$.

Notice that additive representability implies semiadditivity. The converse is not true with more than two issues. For example, the preference $\succ : \{x, y, z\} \succ \{x, y\} \succ \{x, z\} \succ \{y, z\} \succ \{z\} \succ \{y\} \succ \{x\} \succ \emptyset$ is semiadditive but not additively representable. Also notice that whenever \succ is semiadditive, $M(\succ) = G(\succ)$: the best set is the set of all good issues. We can now state:

THEOREM 1': A voting scheme $f : (P^{SA})^n \rightarrow 2^K$ satisfies anonymity, neutrality, voter sovereignty, and is strategy-proof on P^{SA} if and only if it is voting by quota.

Thus, if we enlarge the domain restriction from additively representable to semiadditive preferences, voting by quota will still be strategy-proof on the new domain, and the only method to have this and all other properties of Theorem 1. In Section 3 we will show that no further domain enlargement of this kind could lead to a similar result.

Next, we extend our results to nonanonymous voting schemes.

Definition 8. A committee (monotonic simple game) is a pair $C = (N, W)$, where $N = \{1, \dots, n\}$ is the set of voters, W is a nonempty set of nonempty coalitions of N , which satisfies $[S \in W \ \& \ T \supset S] \rightarrow T \in W$.

Coalitions in W are called winning coalitions for this committee. $S \in W$ is a minimal winning coalition if and only if $T \subsetneq S \rightarrow T \notin W$ (where $T \subsetneq S$ means $T \subset S$ and $T \neq S$). Let $C_Q = (N, W_Q)$ be the committee where $W_Q = \{S \subset K \mid \# S \geq Q\}$. Then the definition of voting by quota Q could be rephrased as follows: f is voting by quota Q if and only if $x \in f(>_1, \dots, >_n)$ whenever $\{i \mid x \in G(>_i)\} \in W_Q$. In a similar fashion, we can define a larger class of voting schemes, based on any given committee.

Definition 9. The scheme $f : P^n \rightarrow 2^K$ is voting by committee, if there exists a committee $C = (N, W)$ such that for all issues $x \in k$ and all profiles $(>_1, >_2, \dots, >_n)$, $x \in f(>_1, >_2, \dots, >_n)$ if and only if $\{i \mid x \in G(>_i)\} \in W$.

Many actual decision processes can be represented by voting by committee but not voting by quota. This will typically be the case when decisions are taken by sequential votes involving different groups of decision-makers. We provide two examples of committees leading to voting schemes which are not voting by quota.

(a) (N, W_ω) , with $W_\omega = \{M \mid \sum_{i \in M} \omega_i \geq Q\}$, for given $Q, \omega_1, \omega_2, \dots, \omega_n$.

(b) (N, W_T) , where $N = \{1, 2, 3, 4\}$ and W_T is generated by the minimal winning coalitions $\{1, 2\}$ and $\{3, 4\}$.

The following theorems provide the announced extension of our previous results.

THEOREM 2. A voting scheme $f : (P^A)^n \rightarrow 2^K$ satisfies neutrality, voter sovereignty and is strategy-proof on P^A if and only if it is voting by committee.

THEOREM 2'. A voting scheme $f : (P^{SA})^n \rightarrow 2^K$ satisfies neutrality, voter sovereignty and is strategy-proof on P^{SA} if and only if it is voting by committee.

3. PROOFS OF THE THEOREMS.

It is easy to check that voting by quota and voting by committee satisfy all the conditions of the theorems on the domains $(P^A)^n$ and $(P^{SA})^n$. To prove the converse, we begin by establishing Theorem 2. After that, proving Theorem 2' only requires to show that voting by committee is still the only method to satisfy these conditions on

semiadditive preferences. Theorems 1 and 1' are corollaries of Theorems 2 and 2'; under each of the domain restrictions the additional requirement of anonymity implies that all minimal winning coalitions must be of the same size Q and all coalitions of this size or larger must be winning: this number Q is the quota.

Proof of Theorem 2:

Suppose that $> \in P^A$ is represented by $v : K \rightarrow R$. Since v is defined by its values on K , we can identify $>$ with a vector $v = (v^1, v^2, \dots, v^n) \in R^k$, where v^i is the utility of issue i . (The v^i 's must satisfy $\sum_S v^i \neq \sum_T v^i$ whenever $S \neq T$, and this is assumed throughout.) With this notation, a voting scheme f on $(P^A)^n$ is a function with domain contained in R^{kn} and codomain 2^K , where f is required to take on the same value for any pair of elements of the domain that represent the same preference. The point $v = (v_1, v_2, \dots, v_n) = (v_1^1, v_1^2, \dots, v_1^k; \dots; v_n^1, v_n^2, \dots, v_n^k) \in R^{kn}$ in the domain of f is the n -tuple of k -vector's that represent preferences of all voters. We will call it a profile throughout.

For $v = (v^1, v^2, \dots, v^k)$ and $l \in K$, define $I(v, l) = \{ (w^1, w^2, \dots, w^k) \mid w^i = v^i \text{ for all } i \neq l, \text{ and } w^l > v^l \}$. The set $I(v, l)$ is the collection of utility functions that can be obtained from v by increasing the utility of the l^{th} issue. Define $D(v, l) = \{ w \mid v \in I(w, l) \}$. The set of utility functions which can be connected to v by a sequence of utility increasing transformation applied to issues of $A \subset K$ is defined by $I(v, A)$. Formally $w \in I(v, A)$, if there exist v_1, v_2, \dots, v_p and $a_1, a_2, \dots, a_{p+1} \in A$ such that $v_1 \in I(v, a_1)$, $v_2 \in I(v_1, a_2)$, $\dots, v_p \in I(v_{p-1}, a_p)$ and $w \in I(v_p, a_{p+1})$. With the obvious interpretation, we define $D(v, A) = \{ w \mid v \in I(w, A) \}$.

Proposition 1. If $G(v) = G(w) = S$ for some $S \subset K$ and utility functions $v = (v^1, v^2, \dots, v^k)$ and $w = (w^1, w^2, \dots, w^k)$, then there exist $\hat{v} \in I(v, S)$ and $\check{v} \in D(\hat{v}, K \setminus S)$ such that \check{v} and w represent the same preference.

Proof: The proof involves an easy construction. We just sketch it. Let $\check{v} = t v$, where the scalar t is chosen so large that all the components of \check{v} have larger absolute values than any of the components of v . Now let $\hat{v}^l = \check{v}^l$ if $v^l > 0$, $\hat{v}^l = v^l$ if $v^l < 0$. The reader may check that all the statements in the Proposition are satisfied. I

Proposition 2. Assume that $f(v_1, v_2, \dots, v_n) = B$, and $f(w_1, v_2, \dots, v_n) = B'$, where $w_1 \in I(v_1, l)$. If $B \neq B'$, then $l \in B' \setminus B$.

Proof: Since f is strategy-proof,

$$(a) \quad \sum_{h \in B} v_1^h > \sum_{h \in B'} v_1^h, \text{ and}$$

$$(b) \quad \sum_{h \in B'} w_1^h > \sum_{h \in B} w_1^h.$$

Summing (a) and (b), we get

$$\sum_{h \in B \setminus B'} (w_1^h - v_1^h) > \sum_{h \in B \setminus B'} (w_1^h - v_1^h) \geq 0,$$

The strict inequality implies $\ell \in B \setminus B'$. I

Observe that Proposition 2 is equivalent to that $\ell \in B \setminus B'$ whenever $f(v_1, v_2, \dots, v_n) = B$, $f(w_1, w_2, \dots, w_n) = B'$, $w_1 \in D(v_1, \ell)$, and $B \neq B'$. The following Corollary is immediate.

Corollary A. If $f(v_1, v_2, \dots, v_n) = B$, and $w_i \in I(\hat{v}_i, B)$, while $\hat{v}_i \in D(v_i, K \setminus B)$, for all $i \in N$, then $f(w_1, w_2, \dots, w_n) = B$.

The Corollary can be said in words: increasing the utilities of the issues already chosen and decreasing the utilities of the issues not chosen could not change the outcome.

Proposition 3. If for all i , $G(v_i) = S$, then $f(v_1, v_2, \dots, v_n) = S$.

Proof: By voter sovereignty (VS) we can choose a $w = (w_1, \dots, w_n)$ such that $f(w_1, \dots, w_n) = S$. Starting from w , we can increase the utilities of all issues in S , until they are all positive, and decrease the utilities of all issues not in S , until they are all negative. Call the new profile \hat{w} . By construction, $G(\hat{w}_i) = S$ for all i . Also, by Proposition 2 and its Corollary, $f(\hat{w}) = f(w) = S$.

This already establishes that there must be one profile \hat{w} where S is top for all agents and S is the outcome. To complete the proof, let v be any other profile where S is top for all agents. Proposition 1 allows us to pass from \hat{w} to v by a succession of changes, each of which involves increasing the utilities of issues in S or decreasing the utilities of elements not in S . But then, by Proposition 2, we have $f(v) = f(\hat{w}) = S$.

Define $G(v_1, v_2, \dots, v_n) = \bigcup_{i=1}^{i=n} G(v_i)$; these are the issues that at least some voter likes.

Proposition 4. For any profile (v_1, v_2, \dots, v_n) , $f(v_1, v_2, \dots, v_n) \subset G(v_1, v_2, \dots, v_n)$.

Proof: If the proposition is not true, then there exists a profile (v_1, v_2, \dots, v_n) and an issue l such that $l \in f(v_1, v_2, \dots, v_n)$ but $l \notin G(v_1, v_2, \dots, v_n)$. Construct (w_1, w_2, \dots, w_n) as follows:

- (i) if $v_i^h < 0$, for $h \in f(v_1, v_2, \dots, v_n) \setminus \{l\}$, then let $w_i^h > 0$,
- (ii) if $v_i^h > 0$, for $h \notin f(v_1, v_2, \dots, v_n)$, then let $w_i^h < 0$, and
- (iii) $w_i^h = v_i^h$, otherwise.

By Corollary A,

$$f(w_1, w_2, \dots, w_n) = f(v_1, v_2, \dots, v_n).$$

But this leads to a contradiction, since $G(w_i) = f(v_1, v_2, \dots, v_n) \setminus \{l\}$ for all i , and so by Proposition 3,

$$f(w_1, w_2, \dots, w_n) = f(v_1, v_2, \dots, v_n) \setminus \{l\}. \quad \text{I}$$

We say that a coalition of voters $E \in N$ is winning for $A \subset K$, $A \neq \emptyset$, if there exists a profile (v_1, v_2, \dots, v_n) such that $G(v_i) = A$, for all $i \in E$; $G(v_i) = \emptyset$, for all $i \notin E$, and $f(v_1, v_2, \dots, v_n) = A$.

Proposition 5: Given any $A \subset K$, $A \neq \emptyset$, if W_A denotes the set of all winning coalitions, then $C_A = (N, W_A)$ is a committee.

Proof: By Proposition 3, we know that N is winning for A , and that \emptyset is not winning for A . Hence W_A is a nonempty set of nonempty coalitions. Using Corollary A, it is easy to see that W_A satisfies the monotonicity property. I

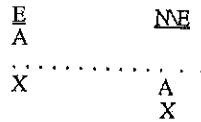
The proof for Theorem 2 will proceed by analysing the structure of C_A . Next we prove that, in fact, C_A is independent of A .

Proposition 6: If E is winning for A , then E is winning for any $B \subset K$.

Proof: We first observe that if E is winning for A , then for any profile (w_1, w_2, \dots, w_n) such that $G(w_i) = A$ for all $i \in E$, and $G(w_i) = \emptyset$ for all $i \notin E$, we must have $f(w_1, w_2, \dots, w_n) = A$. To see this, for each i transform v_i to w_i as follows: for each $i \in E$ apply the transformation in Proposition 1, and for $i \notin E$ go directly from v_i to w_i . The outcome will not change by these transformations for voters in E by Corollary A. To

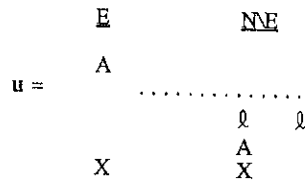
see that it will not change with the transformations for voters in $N \setminus E$, observe that, by Proposition 3, the outcome must remain a subset of A , so when a change occurs it must become a proper subset $A' \subsetneq A$, which means that f is manipulable by someone in $N \setminus E$.

Before we proceed, it is useful to introduce the following notation:

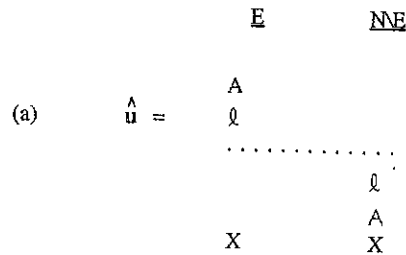


This describes a profile (v_1, v_2, \dots, v_n) , where $G(v_i) = A$ for all $i \in E$, $G(v_i) = \emptyset$ for all $i \notin E$, and X represents the set of issues not in A . Voters may have different preferences among subsets of A and among subsets of X , but in this notation, if any subset A is listed above another subset X , then each of the issues of A (as a singleton) is ranked above each of the issues of X . The dotted line indicates that issues listed above it are good, and issues below it are bad.

(I) First, we will prove the proposition for any $B \supset A$. It is sufficient to establish the result for $B = A \cup \{\ell\}$, where $\ell \in K \setminus A$. Since E is winning for A , $f(u) = A$, where



Consider $f(\hat{u})$, where



By Proposition 4, $f(\hat{u})$ can be exactly one of the following sets:

- (i) $A', A' \subsetneq A,$
- (ii) $A' \cup \{\ell\}, A' \subsetneq A;$
- (iii) $A,$ or
- (iv) $A \cup \{\ell\}.$

To establish (I) it is sufficient to show that the first three cases are impossible. Case (i) is impossible by Corollary A. Next, we show that (ii) is impossible. If not, start with \hat{u} and move ℓ below \emptyset for voters in E , one by one. The outcome will always be $A \cup \{\ell\}$ unless eventually it becomes $f(u) = A$. The scheme f is manipulable at the profile preceding the one where the outcome first becomes A . (This is referred to as a chain argument; it is frequently used in the literature on strategy-proof rules and in what follows). Next, we will show that (iii) is impossible.

If (iii) is true, then $f(\hat{u}) = A$. Then, by Corollary A,

$$f \begin{bmatrix} \begin{matrix} E & NE \\ A & A \\ \ell & \\ \dots & \dots & \dots \\ & \ell \\ X & X \end{matrix} \end{bmatrix} = A$$

Consider

$$v = \begin{bmatrix} \begin{matrix} E & NE \\ \ell & A \\ A & \\ \dots & \dots \\ & \ell \\ X & X \end{matrix} \end{bmatrix}$$

Again by Proposition 4, $f(v)$ is one of the following sets: (1) $A', A' \subsetneq A$; (2) $A \cup \{\ell\}$; (3) $A' \cup \{\ell\}, A' \subsetneq A$; or (4) A . The first case is impossible, as before. Since f is strategy-proof, the second case is impossible by a chain argument. To see that (3) is impossible, observe that if (3) holds, then by Corollary A

$$f \begin{array}{c|c} \underline{E} & \underline{N \setminus E} \\ \hline \varrho & A \\ A & \varrho \\ \dots & \dots \\ X & X \end{array} = A' \cup \{\varrho\},$$

which contradicts Proposition 3. Therefore, $f(v) = A$, and we continue the proof that (iii) is not possible. Since $A \neq \emptyset$, we may write $A = A_1 \cup \{h\}$ and rearrange v to form \hat{v} . We then obtain

$$(b) \quad f(\hat{v}) = f \begin{array}{c|c} \underline{E} & \underline{N \setminus E} \\ \hline \varrho & A_1 \\ A_1 & h \\ h & \\ \dots & \dots \\ & \varrho \\ X & X \end{array} = A_1 \cup \{h\} = A.$$

On the other hand, applying the same decomposition to \hat{w} in (a), and recalling $f(\hat{w}) = A$, we have $f(\hat{w}) = A_1 \cup \{h\}$, where

$$\hat{w} = \begin{array}{c|c} \underline{E} & \underline{N \setminus E} \\ \hline h & \\ A_1 & \\ \varrho & \\ \dots & \dots \\ & \varrho \\ & h \\ X & A_1 \\ & X \end{array}$$

Interchange ϱ and h in \hat{w} and apply neutrality to get the following

$$(c) \quad f(\hat{w}) = f \begin{array}{c|c} \underline{E} & \underline{N \setminus E} \\ \hline \ell & A_1 \\ A_1 & \\ h & \\ \dots & \dots \\ \dots & \dots \\ & h \\ & \ell \\ X & X \end{array} = A_1 \cup \{\ell\}.$$

But (b) and (c) imply that f is manipulable by someone in $N \setminus E$ by a chain argument. (For voters in $N \setminus E$, move h up in \hat{w} , one by one. By Corollary A (applied twice), when the outcome changes it will be $A_1 \cup \{h\}$.) This completes the demonstration that (iii) is impossible.

(II). Next we prove the proposition for $B \subset A$, $B \neq \emptyset$. Again it is sufficient to prove it for $B \subset A$ such that $B \cup \{\ell\} = A$ for some ℓ . Since E is winning for A ,

$$f(v) = f \begin{array}{c|c} \underline{E} & \underline{N \setminus E} \\ \hline B & \\ \ell & \\ \dots & \dots \\ & B \\ & \ell \\ X & X \end{array} = B \cup \{\ell\},$$

and we can choose v so that $v_i^b > 2$ for all $i \in E$ and $b \in B$. Now move ℓ down for all $i \in E$ and form w so that $w_i^b = v_i^b$ for all $i \in E$, $b \in B$, $0 > w_i^\ell > -1$, and $w_i = v_i$ for all $i \notin E$. Thus

$$w = \begin{array}{c|c} \underline{E} & \underline{N \setminus E} \\ \hline B & \\ \dots & \dots \\ \ell & B \\ & \ell \\ X & X \end{array}$$

By Proposition 4, $f(w) \subset B$, and so it is either $B' \subsetneq B$ or B . The first is impossible because it implies that f is manipulable by someone in E (apply a chain argument). But $f(w) = B$ means that E is winning for B .

(III). Finally, E is winning for A implies that E is winning for K by (I) and in turn, E is winning for any subset B of k by (II). I

As a result of Proposition 6, we can say that a coalition of voters E is a winning coalition without referring to any specific subset of K . By Proposition 5, we know that $C = (N, W)$, where W is the set of winning coalitions, is a committee.

Proposition 7. If $\ell \in f((v_1, v_2, \dots, v_n))$, then $M_\ell = \{i \mid v_i^\ell > 0\}$ is a winning coalition.

Proof: Let $M_\ell = \{i \mid v_i^\ell > 0\} \subset N$, and $A = f(v_1, v_2, \dots, v_n) \setminus \{\ell\} \subset K$; if $A = \emptyset$, then $f(\tilde{v}) = \ell$, where \tilde{v} is obtained from v by making $\tilde{v}_i^h < 0$ for $h \neq \ell$. This does not change the outcome and so M_ℓ is winning. If $A \neq \emptyset$, then we can transform v to \hat{v} without changing the outcome:

$$f(\hat{v}) = f \begin{array}{cc} M_\ell & N \setminus M_\ell \\ \left[\begin{array}{cc} \ell & \\ A & A \\ \dots & \dots \\ & \ell \\ X & X \end{array} \right] & = f(v) = A \cup \{\ell\} . \end{array}$$

Furthermore, since f is strategy-proof

$$f(w) = f \begin{array}{cc} M_\ell & N \setminus M_\ell \\ \left[\begin{array}{cc} \ell & \\ A & A \\ \dots & \dots \\ X & X \\ & \ell \end{array} \right] & = A \cup \{\ell\} . \end{array}$$

To see this note that if the outcome changes when we move ℓ in \hat{v} down one by one, then it must become A, and in this case f is manipulable. We can assume that for any $i \in N \setminus M_\ell$, $w_i^\ell < -2k$, $|w_i^h| < 1$, for all $h \neq \ell$. Then every voter in $N \setminus M_\ell$ will prefer any subset A' of A to $A \cup \{\ell\}$. Thus applying a chain argument and recalling Proposition 4, we have

$$f \begin{array}{c} \begin{array}{cc} M_\ell & N \setminus M_\ell \end{array} \\ \left[\begin{array}{cc} \ell & \\ A & \\ \dots & \dots \\ X & X \\ & \ell \end{array} \right] = A' \cup \{\ell\} . \end{array}$$

It is not hard to establish that $A' = A$, and so M_ℓ is winning; however, a simpler route to this conclusion is to move $h \in A \setminus A'$ below \emptyset for $i \in M_\ell$. I

Proposition 8. Given any profile $v = (v_1, v_2, \dots, v_n)$, if for all $\ell \in G(v)$, $M_\ell = \{i | v_i^\ell > 0\}$ are minimal winning coalitions in $C = (N, W)$, then

$$f(v_1, v_2, \dots, v_n) = G(v).$$

Proof: The argument is by induction.

(I). If $\#G(v) = 1$, then the proposition is true by the definition of $C = (N, W)$.

(II). Assume it is true for all (v_1, v_2, \dots, v_n) such that $\#G(v) = m+1$. If the profile v is of the form $M_\ell = M_h$, for all $\ell, h \in G(v)$, then again the conclusion follows from the definition of $C = (N, W)$. Thus, if the proposition is not true, then there exist issues ℓ and k and a profile w such that $\ell \notin f(w)$, $w_j^k > 0$, and $j \in N \setminus M$, where $M = \{i | w_i^\ell > 0\}$. We assume further that

$$(*) \quad w_j^\ell + \sum_{h \in G(w) \setminus \{k, \ell\}} w_j^h < \sum_{h \in f(w)} w_j^h .$$

Now transform w to \hat{w} by requiring $\hat{w}_j^k < 0$; $\hat{w}_j^h = w_j^h$, $h \neq k$; and $\hat{w}_i = w_i$, $i \neq j$. We now show $f(\hat{w}) = G(w) \setminus \{k\}$. This is true because (i) $k \notin f(w)$ by Proposition 7; (ii) successive moves of k down in \hat{w} below zero will not change the outcome by Corollary A, and (iii) the resulting profile, call it \tilde{w} , satisfies the conditions of the proposition for $\#G(\tilde{w}) = m$. Finally by the induction hypothesis $f(\hat{w}) = f(\tilde{w}) = G(\tilde{w}) \setminus \{k\}$. It follows from (*) that f is manipulable at \hat{w} , a contradiction. Thus, the proposition is also true for (v_1, v_2, \dots, v_n) such that $\#G(v) = m+1$. I

Proposition 9. For any profile (v_1, v_2, \dots, v_n) and any $\ell \in K$, if $M_\ell = \{i \mid v_i^\ell > 0\} \in W$, then $\ell \in f(v_1, v_2, \dots, v_n)$.

Proof: The result follows immediately from Proposition 2 and Proposition 8. I

The claim of Proposition 7 plus that of Proposition 9 is equivalent to the statement that f is voting by committee. Therefore we complete the proof of Theorem 2. I

Proof of Theorem 2'.

Again, it is easy to show that voting by committee satisfies the conditions on $(PSA)^n$. Now we have to show that it is the only method to do so.

Let f_A be the restriction of f to $(P^A)^n$. By Theorem 2, f_A is voting by committee for some $C = (N, W)$. Denote by f^C voting by committee G on $(PSA)^n$. For $S \subset N$, denote by f_S the restriction of f to $(PSA)^S \times (P^A)^{N-S}$. In particular, $f_{\{i\}}$ stands for the restriction of f to the domain where i 's preferences are semiadditive and anybody else's are additive.

Theorem 2' will be proved if we can show that $f_N = f^C$. To prove it, we use an induction argument on the size of S . We have already argued that $f_\emptyset = f^C$. Assume that $f_T = f^C$ for all T of a certain size or less. We want to prove that, for $i \in N \setminus T$, $f_{T \cup \{i\}} = f^C$.

For a given preference profile $\succ = (\succ_T, \succ_i, \succ_{N \setminus (T \cup \{i\})})$ in $(PSA)^T \times (P^A)^{N \setminus T}$, we consider any additively representable preferences $\bar{\succ}_i$ such that $G(\bar{\succ}_i) = G(\succ_i)$. Denote $\bar{\succ} = (\succ_T, \bar{\succ}_i, \succ_{N \setminus (T \cup \{i\})})$. By the induction hypothesis, $f(\bar{\succ}) = f^C(\bar{\succ})$, which is independent of the particular $\bar{\succ}_i$, as long as $G(\bar{\succ}_i) = G(\succ_i)$ holds. We claim:

(I) $f(\bar{y}) \cap G(\succ_i) \subset f^C(\bar{y})$. This is because for any issue $l \in f(\succ) \cap G(\succ_i)$, we can construct \bar{y}_i as follows: $u_i^l = 2k$, $|u_i^j| < 1, j \neq l$, and $G(\bar{y}_i) = G(\succ_i)$ so that voter i with preference \bar{y}_i prefers any subset of issues that includes l to those which do not. The strategy-proofness of f implies $l \in f^C(\bar{y})$.

(II) $f^C(\bar{y}) \cap B(\succ_i) \subset f(\succ)$. In this case, for any $l \in f^C(\bar{y}) \cap B(\succ_i)$ we construct \bar{y}_i as follows: $u_i^l = -2k$, $|u_i^j| < 1, j \neq l$, and $G(\bar{y}_i) = G(\succ_i)$. So voter i with preference \bar{y}_i prefers any subset of issues that excludes l to those which do not. Again the strategy-proofness of f implies $l \in f(\succ)$.

What (I) says is that $f^C(\bar{y})$ contains at least those "good" issues that $f(\succ)$ contains, and (II) says that $f^C(\bar{y})$ contains no more "bad" issues than $f(\succ)$. So if $f^C(\bar{y}) \neq f(\succ)$, then $f^C(\bar{y})$ will be strictly preferred to $f(\succ)$. But then f is manipulable at \succ by voter i . Hence $f(\succ) = f^C(\bar{y})$. This completes the proof of Theorem 2'. I

3. SEMIADDITIVE PREFERENCES AND THE STRATEGY-PROOFNESS OF VOTING BY COMMITTEE.

We have seen that voting by committee is the only method to satisfy our list of properties on the domain on semiadditive preferences. In this section we show that voting by committee cannot be strategy-proof on any larger domain. To make this idea precise, we restrict attention to domain restrictions which allow for a wide enough variety of preferences.

Definition 10. A domain $R = T_1 \times T_2 \times \dots \times T_n \subset P^n$ is rich if, for all $i \in N$ and all $A \subset K$, there exist $\succ_i \in T_i$, such that $A = G(\succ_i)$.

Definition 11. A committee $C = (N, W)$ is dictatorial if there exists $i \in N$ (the dictator) such that

$$S \in W \Leftrightarrow i \in S.$$

Definition 12. Given a committee $C = (N, W)$ a voter i is dummy in C if $i \notin \bigcup_{\alpha} N_{\alpha}$, where the N_{α} 's are the minimal winning coalitions of C .

If a committee C is dictatorial, then, obviously, voting by committee C is strategy-proof on the universal domain. So we are only interested in the case when C is not dictatorial. Since voting by committee C does not depend on the preferences of the dummy voters in C, the requirement that voting by committee C be strategy-proof on a domain R will not impose any restriction on the admissible preferences of these dummy voters. But it does impose restrictions on the admissible preferences of non-dummy voters, which are exactly given by semiadditivity. Before showing that, we first prove a proposition which gives an equivalent specification of PSA.

Proposition 10. Requiring that the preference \succ be semiadditive is equivalent to the condition

$$(**) \quad \text{for all } A \in 2^X, \text{ and } x \notin A, \quad A \cup \{x\} \succ A \text{ if and only if } x \in M(\succ).$$

Proof: The necessity of (**) is obvious. To prove the sufficiency of (**), we first show that it implies $M(\succ) = G(\succ)$. If $x \in M(\succ)$, consider (**) with $A = \emptyset$, then $\{x\} \succ \emptyset$. If $x \notin M(\succ)$ consider (**) with $A = \emptyset$, then $\emptyset \succ \{x\}$. So $M(\succ) = G(\succ)$. Then (**) becomes the definition of semiadditivity when replacing $M(\succ)$ by $G(\succ)$. So $\succ \in \text{PSA}$. I

THEOREM 3. If voting by (a non-dictatorial) committee C is strategy-proof on a rich domain $R = T_1 \times T_2 \times \dots \times T_n$, then for any non-dummy voter i , $T_i \subset \text{PSA}$.

Proof: Given any preference $\succ_i \in T_i$, take any x, A such that $x \notin A$. There are two cases: $x \in M(\succ_i)$, or $x \notin M(\succ_i)$.

(I) $x \in M(\succ_i)$. Because voter i is non-dummy we can find a minimal winning coalition N_1 such that $i \in N_1$, and because C is not dictatorial we can find another winning coalition N_2 such that $i \notin N_2$. Since R is rich, we can choose \succ_j for $j \in N \setminus \{i\}$ so that

$$\begin{aligned} M(\succ_j) &= A \cup \{x\}, & j \in N_1 \cap N_2 \\ M(\succ_j) &= \{x\}, & j \in N_1 \setminus N_2, \\ M(\succ_j) &= A, & j \notin N_2 \setminus N_1 \text{ and} \\ M(\succ_j) &= \emptyset, & \text{Otherwise.} \end{aligned}$$

With preferences defined that way

$$f [M(>_1), M(>_2), \dots, M(>_i), \dots, M(>_n)] = A \cup \{x\}$$

But if voter i announces $E = M(>_i) \setminus \{x\}$

$$f [M(>_1), M(>_2), \dots, E, \dots, M(>_n)] = A$$

f is strategy-proof implies $A \cup \{x\} \succ_i A$.

(II) When $x \notin M(>_i)$, by the same method, we can show that f is strategy-proof implies $A \succ_i A \cup \{x\}$.

Hence, by Proposition 10, $\succ_i \in \text{PSA}$.

I

4. EFFICIENCY AND STRATEGY-PROFNESS ON RESTRICTED DOMAINS.

We now turn to the efficiency properties of voting by committee, and prove that in general the method will not be Pareto optimal even under strong domain restrictions. This will not surprise the reader, as it is just the expression in our framework of the well known trade-off between efficiency and strategy-proofness.

• As an introductory example, consider a situation where $N = \{1, 2\}$, $k = \{x, y\}$, and voting by quota Q applies. If $Q = 1$,

$$f \begin{bmatrix} \{x\} & \{y\} \\ \emptyset & \emptyset \\ \{xy\} & \{xy\} \\ \{y\} & \{x\} \end{bmatrix} = \{xy\}, \text{ and yet } \{xy\} \text{ is Pareto-dominated by } \emptyset.$$

Similarly, if $Q = 2$,

$$f \begin{bmatrix} \{x\} & \{y\} \\ \{xy\} & \{xy\} \\ \emptyset & \emptyset \\ \{y\} & \{x\} \end{bmatrix} = \emptyset, \text{ while } \{xy\} \text{ Pareto-dominates } \emptyset.$$

These examples generalize.

Definition 13. A voting scheme f is Pareto optimal if for all $(\succ_1, \succ_2, \dots, \succ_n)$ in its domain, and for all $B \subset K$, $B \neq f(\succ_1, \succ_2, \dots, \succ_n)$, $f(\succ_1, \succ_2, \dots, \succ_n) \succ_i B$ for at least one i .

Definition 14. A committee $C = (N, W)$ is proper if $S \in W \Rightarrow N \setminus S \notin W$.

Definition 15. A committee $C = (N, W)$ is strong if $S \notin W \Rightarrow N \setminus S \in W$.

Again it is trivial that voting by committee C is Pareto optimal if C is dictatorial. In what follows we consider only non-dictatorial committees. It turns out that voting by committee will be rarely Pareto optimal.

Proposition 11. Assume C is non-dictatorial. Voting by committee C is Pareto optimal if and only if $k = 2$, and C is both proper and strong (In the case of voting by quota, we have $k = 2$ and $2Q = n+1$).

Proof: (I) First, we prove the sufficiency of the conditions. When $k = 2$, denote the two issues by x and y , the final outcome can only be one of the four: $\{x\}$, $\{y\}$, $\{xy\}$, and \emptyset .

(i) If $\{x\}$ is the outcome for a profile $(\succ_1, \succ_2, \dots, \succ_n)$, then $N_x = \{i \mid \{x\} \succ_i \emptyset, i \in N\}$ is a winning coalition. There must be some $i \in N_x$ such that $\emptyset \succ_i \{y\}$, otherwise $\{xy\}$ should be the outcome. Hence $\{x\}$ is a Pareto optimal outcome.

(ii) If $\{y\}$ is the outcome, then, by the same reason, $\{y\}$ is also a optimal outcome.

(iii) If $\{xy\}$ is the outcome, then both $N_x = \{i \mid \{x\} \succ_i \emptyset, i \in N\}$ and $N_y = \{i \mid \{y\} \succ_i \emptyset, i \in N\}$ are winning coalitions. Since C is proper, $N_x \cap N_y \neq \emptyset$. Hence, $\{xy\}$ is a Pareto optimal outcome.

(iv) If \emptyset is the outcome then both M_x and M_y are not losing coalitions. Since C is strong, $N_x \cup N_y \neq N$. Hence, \emptyset is also a Pareto optimal outcome.

That completes the proof of the sufficiency of the conditions.

(II) Now we start to prove the necessity of the conditions.

(i) If C is not proper, then there exist $N_1, N_2 \in W$, such that $N_1 \cup N_2 = N$. Pick $x, y \in K$. Let \succ_1, \succ_2 be the following preferences:

$$\{x\} \succ_1 \emptyset \succ_1 \{xy\} \succ_1 \{y\} \succ_1 \dots, \text{ and}$$

$$\{y\} \succ_2 \emptyset \succ_2 \{xy\} \succ_2 \{x\} \succ_2 \dots$$

For the preference profile where every voter in N_1 has \succ_1 and everyone in N_2 has \succ_2 , voting by committee C gives the outcome $\{xy\}$, which is Pareto dominated by \emptyset .

(ii) If C is not strong, then there are $N_1, N_2 \in W$, such that $N_1 \cup N_2 = N$. Let \succ_1, \succ_2 be

$$\{x\} \succ_1 \{xy\} \succ_1 \emptyset \succ_1 \{y\} \succ_1 \dots, \text{ and}$$

$$\{y\} \succ_2 \{xy\} \succ_2 \emptyset \succ_2 \{x\} \succ_2 \dots$$

For the preference profile where every voter in N_1 has \succ_1 and every voter in N_2 has \succ_2 , voting by committee C leads to the outcome \emptyset , which is dominated by $\{xy\}$.

(iii) If C is both proper and strong then it is easy to verify that no single voter alone can be winning (keep in mind that C is non-dictatorial). As a result, there are at least three voters. Take any minimal winning coalition N_0 . $N_3 = N \setminus N_0$ cannot be empty. Split N_0 into two nonempty coalitions $N_1 \cup N_2$. Thus we get three non-winning coalitions N_1, N_2 and N_3 , such that $N = N_1 \cup N_2 \cup N_3$. Since $k \geq 3$, let \succ_1, \succ_2 , and \succ_3 be

$$\{x\} \succ_1 \{xy\} \succ_1 \{xz\} \succ_1 \{xyz\} \succ_1 \emptyset \succ_1 \dots$$

$$\{y\} \succ_2 \{yz\} \succ_2 \{yx\} \succ_2 \{xyz\} \succ_2 \emptyset \succ_2 \dots,$$

$$\{z\} \succ_3 \{zx\} \succ_3 \{zy\} \succ_3 \{xyz\} \succ_3 \emptyset \succ_3 \dots,$$

For the profile where voters in N_1 have \succ_1 , those in N_2 have \succ_2 , and those in N_3 have \succ_3 , voting by committee C leads to the outcome \emptyset , which is dominated by $\{xyz\}$. \square

Proposition 11 immediately implies the following Theorem.

THEOREM 4. For $k \geq 3$, there exists no voting scheme that is strategy-proof on the additively representable preferences, and satisfies neutrality and Pareto optimality.

6. FURTHER RESULTS AND A FINAL CONJECTURE.

The domain of a voting scheme is a subset of preference n -tuples; in principle, any feature of such preferences is allowed to influence the choice of issues. Yet, our theorems show that voter sovereignty, neutrality and strategy-proofness on additive or semiadditive preferences can only be satisfied by voting by committee, and these

methods only rely on what is the best set of issues for each of the voters. All other information about their preferences, even if available, cannot be used. In fact, requiring that "only tops count" is an attractive simplicity condition. Until now, we have not imposed it; it has been obtained as a consequence of our conditions.

We now restrict attention to the class of voting schemes which depend only on the top sets of issues of all voters' preferences. That is, each voter announces a set of issues and then a set of issues is produced. So the voting schemes are of the following form:

$$f: (2^K)^n \rightarrow 2^K.$$

In this case, we can fully characterize the structure of the voting schemes which are strategy-proof on $(P^A)^n$. The distinct feature of the following result is that it does not impose any condition other than strategy-proofness. In order to make our characterization completely general, we extend slightly our previous definition of a committee. Notice that a strategy-proof scheme may be such that some issue is never elected, or always elected. These (unattractive) cases can be taken care of if we allow for a generalized notion of a committee, admitting an empty set of winning coalitions, or the empty set as a winning coalition.

Definition 16. A generalized committee is a pair $C = (N, W)$, where $N = \{1, \dots, n\}$ is the set of voters, W is a set of coalitions of N , which satisfies $[S \in W \text{ and } T \supset S] \rightarrow T \in W$.

We can now state:

THEOREM 5. A voting scheme $f: (2^K)^n \rightarrow 2^K$ is strategy-proof on $(P^A)^n$ if and only if, for each issue l in K , there is a generalized committee $C_l = (N, W_l)$ such that $l \in f(S_1, S_2, \dots, S_n) \Leftrightarrow N_l = \{i \in N \mid \text{voter } i \text{ considers } l \text{ "good"}\}$ is a winning coalition in C_l .

Proof: (I) We first notice that if $l \in f(S_1, S_2, \dots, S_i, \dots, S_n)$, and $l \in S_i$, then for any T_i , $l \in T_i$ implies $l \in f(S_1, S_2, \dots, T_i, \dots, S_n)$. That is because for any T_i we can find a $v_i \in P^A$, such that $G(v_i) = T_i$, $v_i^l > 2k$, and $|v_i^j| < 1$, for $j \neq l$. Then

voter i with preference v_i prefers $f(S_1, S_2, \dots, S_i, \dots, S_n)$ to any set of issues excluding l . Then f is strategy-proof implies $l \in f(S_1, S_2, \dots, T_i, \dots, S_n)$.

(II) Second, if $l \in f(S_1, S_2, \dots, S_i, \dots, S_n)$, and $l \notin S_i$, then for any T_i , $l \in f(S_1, S_2, \dots, T_i, \dots, S_n)$. That is because we can find $v_i \in P^A$ such that $G(v_i) =$

$S_i, v_i^l < -2k, |v_i^j| < 1$, for all $j \neq l$. Thus voter i with preference v_i prefers any set of issues excluding l to $f(S_1, S_2, \dots, S_i, \dots, S_n)$. f is strategy-proof then implies $l \in f(S_1, S_2, \dots, T_i, \dots, S_n)$.

(III) Now, for any issue l denote $W_l = \{M_l \subset N \mid \text{there exists an } S = (S_1, S_2, \dots, S_n) \text{ such that } M_l = \{i \mid l \in S_i\}, \text{ and } l \in f(S)\}$. We show that $C_l = (N, W_l)$ is a generalized committee. Suppose $M_l \in W_l$. Find $S = (S_1, S_2, \dots, S_n)$ such that $M_l = \{i \mid l \in S_i\}$, and $l \in f(S)$. For any $M \supset M_l$, construct S' as follows: $S'_i = K$, if $i \in M$; $S'_i = S_i$, otherwise. Using (i), (ii) and a chain argument, we conclude $l \in f(S')$. But $M = \{i \mid l \in S'_i\}$, so $M \in W_l$. This proves that $C_l = (N, W_l)$ is a generalized committee.

(iv) Finally, we have to show that for any l and $S = (S_1, S_2, \dots, S_n)$, $l \in f(S)$ if and only if $M_l = \{i \mid l \in S_i\} \in W_l$. The "only if" part is just the definition of C_l . On the other hand, if the latter is true, then there is $S' = (S'_1, S'_2, \dots, S'_n)$, $l \in S'_i \Leftrightarrow l \in S_i$, and $l \in f(S')$. Using (i), (ii) and a chain argument, we conclude that $l \in f(S)$. I

As a final comment, we want to conjecture that a result similar to Theorem 5 could be obtained for the general class of voting schemes satisfying voter sovereignty without imposing the "tops only" simplicity assumption. An obvious route would be to prove directly that this requirement is an implication of strategy-proofness and voter sovereignty.

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